TITLE: Unified TSP™ Index Return and Price Algorithms

INVENTORS: John Wang, Calvin Miao, Jing Li, Fan Zhang

USPTO Patent Application Number: 62440717

BACKGROUND

In the US financial markets residential loans are typically organized by borrower rate, rate type, and loan term for the purpose of securitization into mortgage backed securities (MBS). In addition, certain MBS, similar in rate, rate type, and loan term are deemed "more desirable" than others. These mortgage(s) or mortgage pools are commonly referred to as "specified pools." The specified pools, like other MBS, are bought and sold in the "over the counter" bond market. However, due to their nature they often command a premium or "pay-up" relative to TBA eligible MBS. The ability to determine the market value and appropriate premium applied to specified pools in a transparent and efficient manner is important to all market participants involved in creating, buying, selling, servicing, and holding these securities. Mortgage loan originators, servicers, underwriters, issuers, trustees, investment bankers, fund managers, and broker-dealers all require tools and methods to determine the implied value of the mortgage backed securities they are trading, structuring, and holding.

The combination of methodologies including:

- R-score (The R-score ranks the voluntary prepayment behavior),
- D-score (The D-score ranks the involuntary prepayment behavior)

Which together make it possible to understand the risk factor level of specified pool MBS vis-à-vis those MBS pools deemed less desirable thereby facilitating the proper premium assignment to specified pool MBS and calculation of risk adjusted yield which allows traders and investment managers to accurately assess the value of these pools. Further, the combination of methodologies act to define the convexity profile of the loan pool with respect to both voluntary and involuntary prepayment risk.

The specified pool MBS market can now be assessed properly under the geometric and mathematical framework which thoroughly explains the pricing relationship between those MBS identified as specified pools versus the outstanding tradeable universe of MBS pools.

TRADEABLE SPECIFIED POOL INDEX™

The Tradeable Specified Pool Index™, is a mortgage backed securities tracking index that market participants can use to track the composition and performance of the tradeable specified pool population of fixed rate MBS issued and guaranteed by Fannie Mae (FNMA), Freddie Mac (FHLMC), and Ginnie Mae (GNMA). The Tradable Specified Pool Index is designed so that market participants will have a practical, real-world way to track the specified pool MBS market with an index which reflects the portion of the specified pool market outstanding that are not otherwise encumbered. By CMO holdings, Super Pool Holdings (Mega, Giant, Platinum), and Federal Reserve portfolio holdings. As of Nov, 2016, there are about 300,000 outstanding Tradeable Specified Pools.

INDEX SECURITIES INCLUSION/EXCLUSION CRITERA

Currency Denomination:	All securities' par values, principal, and interest are exclusively in U.S. Dollars.
Index Pool Composition:	Tradeable Specified Pool Index constituents will be selected from fixed-rate U.S. Agency MBS pools that are recognized by market participants as "specified pools."

- Quality – US agency MBS should continue to have least the same rating as U.S. Sovereign Debt.

- Coupon – Fixed-rate coupons should be in 50 basis points increments in accordance with standardized pooling practices.

- Security Types –
    1. GNMA I & II, FHLMC, FNMA fixed

rate MBS
2. Whole and portions of pools recognized as specified pools

- **Exclusions** – those pools not meeting the following categorizations.
    1. **Low Loan Balance**: The maximum original loan size at issuance <= 85k
    2. **Moderate Loan Balance**: The maximum original loan size at issuance > 85k and <= 110k
    3. **High Loan Balance:** The maximum loan size at issuance in the pool > 110k and <= 175k
    4. **Investor:** The pool reported occupancy type is 100% investor
    5. **High Loan-to-Value:** The pool minimum loan-to-value is >= 81
    6. **New York:** The state concentration is 100% New York.
    7. **Puerto Rico:** The state concentration is 100% Puerto Rico.
    8. **Texas:** The state concentration is 100% Texas.
    9. **Illinois:** The state concentration is 100% Illinois.
    10. **Florida:** The state concentration is 100% Florida.
    11. **Louisiana:** The state concentration is 100% Louisiana.
    12. **NY-TX-FL:** The state concentration is New York, Texas, and Florida only.
    13. **NJ-TX-PA:** The state concentration is New Jersey, Texas, Pennsylvania only.
    14. **TX-FL:** The state concentration is Texas and Florida only.

15. **TX-LWAC:** state concentration is 100% Texas and max wac in pool coupon is <=3/8
16. **HSATO:** min SATO in pool >= 5/8
17. **PR 150:** The state concentration is 100% PR and the max loan size is 150k.
18. **NY150:** The state concentration is 100% New York and the max loan size is 150k.
19. **FL150:** The state concentration is 100% Florida and the max loan size is 150K.
20. **TX150:** The state concentration is 100% Texas and the max loan size is 150k.
21. **LA 150:** The state concentration is 100% Louisiana and the max original loan size <= 150k
22. **LWAC:** Max weighted average coupon in pool – coupon is <=3/8.
23. **LFICO:** The max FICO in the pool <= 680.
24. **HHLTV:** The min orig. ltv in the pool > 95.
25. **LFICO_HLTV:** The max FICO in the pool is <= 680 and the min orig. LTV >=81.
26. **LFICO_HLB:** The max FICO in the pool <= 680 and the max orig. loan size in the pool > 110k and max orig. loan size <= 150k.
27. **LFICO2:** The current orig. FICO < 675 and max loan size >150
28. **HLTV2:** Current orig. LTV > 85 and max loan size > 150

Index Frequency: Index level and returns will be calculated at the end of each month using actual cash flows and composition changes that occur daily

	during each calculation period.
Reinvestments:	Intra-month security cash flows contribute to monthly index return and are reinvested into the index for calculations at the end of each month to reflect total returns over each successive monthly period.

INDEX RETURN CALCULATION METHODOLOGY

Changes in the Index will reflect the aggregate total return of the underlying pools' constituent pools' total returns over a monthly observation period. For the Tradeable Specified Pool Index, the three components of total return that are aggregated for each security will comprise of the following:

- Price Change Return: the difference in price from the beginning of the observation period versus that of the ending will constitute the total return due to price change.
  - Price Change Return =

    $$\frac{End\ Period\ Price - Begin\ Period\ Price}{Begin\ Period\ Price + Accrued\ Interest}$$

- Coupon Interest Return: The amount of Coupon interest accrued/paid during the observation period based on the beginning period security balance will constitute the total return coupon interest component.

- Coupon Interest Return =

  $$\frac{(Security\ Coupon/360) * Num\ days\ in\ period}{Begin\ Period\ Price + Accrued\ Interest}$$

- Principal Paydown Return: The gain/loss from monthly scheduled and unscheduled principal payments that occur when the security price is trading above or below par.
  - Principal Paydown Return =

  $$\frac{(1 - survial\ facotr) * (100 - End\ Period\ Price - \frac{Coupon}{360} * days\ in\ period)}{Begin\ Period\ Price + Accrued\ Interest}$$

BRIEF SUMMARY OF R-score AND D-score

We use the R-score and the D-score in addition to the market price and yield information to construct the Tradeable Specified Pool Index. A forecasting model for analytics using agency prepayment R-Score and D-Score and associated loan-level or pool-level analytics for Fannie Mae, Freddie Mac, and Ginnie Mae MBS. The R-Score ranks the voluntary prepayment behavior, while the D-Score ranks the involuntary prepayment behavior. The model uses a host of the associated loan-level or pool-level analytics including interest rate, home price index, credit cycle, turnover, refinance, cash-out, buyout, and curtailment. The model also includes forecasting four prepayment vectors of pricing agency bonds:

1. A vector of predicted voluntary prepayment rates
2. A vector of predicted involuntary prepayment rates
3. A vector of the 60+ days delinquency rate
4. A loss severity factor.

The key innovation in the model is the deconvolution of the macro-economic cycles of credit and market risks from the interacting characteristics of individual loan profiles, agency, and servicer treatments, underwriting at different periods of time, and government and housing policies. The data used to develop the scores consist of all agency loans originated from 1991 to 2015. The scores include thirty-one factors, eight key interaction terms, and twenty-one sub-components. Both R-scores and D-scores are calibrated across all sub-components and refreshed monthly. The model uses an innovative non-linear calibration to map all scores between 100 and 999.

DETAILED DESCRIPTION AND BEST MODE OF IMPLEMENTATION

The model is implemented as a set of computer programs running on a massively parallel, memory based ubiquitous computing system that uses a large heterogeneous database of securities data going back several decades along with other inputs such as interest rates, home price indexes, etc. The model can also be used for other types of financial or other modeling using the appropriate data and factors.

DECONVOLUTION MODEL

The model includes eleven vectors

$$\boldsymbol{v} = \left(\boldsymbol{u}_{crr}, \boldsymbol{m}_{crr}, \boldsymbol{a}_{crr}, \boldsymbol{u}_{cdr}, \boldsymbol{m}_{cdr}, \boldsymbol{a}_{cdr}, \boldsymbol{u}_{deq}, \boldsymbol{m}_{deq}, \boldsymbol{a}_{deq}, \boldsymbol{r}_{sev}, \boldsymbol{r}_{cur}\right)$$

Each vector is the sum of individual factors and interaction terms

$$\boldsymbol{v} = \boldsymbol{v}_o + \sum_{i=1}^{n} \boldsymbol{v}_i + \sum_{i,j,\dots,k}^{n} \boldsymbol{f}_{ij\dots k} \boldsymbol{s}_i \boldsymbol{s}_j \dots \boldsymbol{s}_k$$

Each factor is given by, depending on the number $\boldsymbol{n}$ of available factors,

$$\boldsymbol{v}_i = \boldsymbol{x}_i + \boldsymbol{a}_i(\mathbf{1} - \boldsymbol{n}/\boldsymbol{m})$$
$$\boldsymbol{s}_i = \boldsymbol{s}(\boldsymbol{x}_i)$$

The parameter $\boldsymbol{x}_i$ specifies the complete-factor ($\boldsymbol{n} = \boldsymbol{m}$) model, while the parameter $\boldsymbol{a}_i$ specifies adjustment for the partial-factor ($\mathbf{1} \leq \boldsymbol{n} < m$) model. The parameter $\boldsymbol{s}_i$ specifies the interaction terms, with the interaction strength given by the coefficient $\boldsymbol{f}_{ij\dots k}$.

R & D SCORES

The R-score is based on the average of the model vector $\boldsymbol{u_{crr}}$ over the next twelve months

$$\boldsymbol{R} = \begin{cases} \boldsymbol{R_s \langle u_{crr} \rangle \frac{1 - R_l}{1 - \langle u_{crr} \rangle} + R_o} & \boldsymbol{\langle u_{crr} \rangle < R_l} \\ \boldsymbol{R_s \langle u_{crr} \rangle + P_o} & \boldsymbol{R_l \le \langle u_{crr} \rangle \le R_h} \\ \boldsymbol{R_s \langle u_{crr} \rangle \frac{1 + R_h}{1 + \langle u_{crr} \rangle} + R_o} & \boldsymbol{\langle u_{crr} \rangle > R_h} \end{cases}$$

The D-score is based on the average of the model vector $\boldsymbol{u_{cdr}}$ over the next twelve months

$$\boldsymbol{D} = \begin{cases} \boldsymbol{D_s \langle u_{cdr} \rangle \frac{1 - D_l}{1 - \langle u_{cdr} \rangle} + D_o} & \boldsymbol{\langle u_{cdr} \rangle < D_l} \\ \boldsymbol{D_s \langle u_{cdr} \rangle + D_o} & \boldsymbol{D_l \le \langle u_{cdr} \rangle \le D_h} \\ \boldsymbol{D_s \langle u_{cdr} \rangle \frac{1 + D_h}{1 + \langle u_{cdr} \rangle} + D_o} & \boldsymbol{\langle u_{cdr} \rangle > D_h} \end{cases}$$

The model for R/D Scores also produces the probability of corresponding events, such as prepayment, delinquency, default, etc.

Monthly probability of an event, based on the model, is

$$\boldsymbol{P_m = \frac{1}{1 + exp\left(\frac{S_m - S_0}{S_s}\right)}}$$

where P_m and S_m are the scores for month m respectively, and,

S_0 and S_s are the model parameters for score constant and slope for the collateral types, respectively.

Based the probabilities, we can compute the projected CPRs at months 1, 3, 6, and 12, for the short term forecast.

For longer term forecast, the aging and seasonality curves for the specific collateral types can be merged with the above CPRs.

MANIFOLD VECTORS

The monthly voluntary prepayment rate is given by

$$\boldsymbol{p_{crr} = \frac{e^{-u_{crr}}}{1 + e^{-u_{crr}} + e^{-u_{cdr}}}\left(\frac{e^{m_{crr}}}{e^{a_{crr}}}\right) + (1 - e^{r_{cur}})}$$

The monthly involuntary prepayment rate is given by

$$\boldsymbol{p_{cdr} = \frac{e^{-u_{cdr}}}{1 + e^{-u_{crr}} + e^{-u_{cdr}}}\left(\frac{e^{m_{cdr}}}{e^{a_{cdr}}}\right)}$$

The 60+ day delinquent rate is given by

$$\boldsymbol{p_{deq} = \frac{e^{-u_{deq}}}{1 + e^{-u_{deq}}}\left(\frac{e^{m_{deq}}}{e^{a_{deq}}}\right)}$$

The loss severity factor is given by

$$\boldsymbol{f_{sev} = e^{r_{sev}}}$$

DESCRIPTION TRADEABLE SPECIFIED POOL INDEX

OPTION PRICING MODEL

$$COUPON = \{2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7\} \quad (1)$$

$$AGENCY = \{fh, fn, g1, g2\} \quad (2)$$

$$TERM = \{15, 30\} \quad (3)$$

$$TSP = \{Tradeable\ Specified\ Pool\} \quad (4)$$

$$TPOOL_{ag\_tm\_cp} = \{pool \in TSP | agency = ag, term = tm, coupon = cp\} \quad (5)$$

$$ATOMS_{*} = \{Available\ TBA - Only\ MBS\ Supply\} \quad (6)$$

$$APOOL_{ag\_tm\_cp} = \{pool \in ATOMS | agency = ag, term = tm, coupon = cp\} \quad (7)$$

$$PScoreQ1_{ag\_tm\_cp} \equiv the\ first\ quartile's\ PScore\ for\ pool\ in\ APOOL_{ag\_tm\_cp} \quad (8)$$

$$QPOOL_{ag\_tm\_cp} = \left\{pool \in POOL_{ag_{tm\_cp}} | PScore < PScoreQ1_{ag\_tm\_cp}\right\} \quad (9)$$

$$w_{qplatc} = \frac{currBal}{\sum_{platc \in QPOOL_{ag\_tm\_cp}} currBal} \quad (10)$$

$$aggPScore_{ag\_tm\_cp} = \sum_{platc \in QPOOL_{ag\_tm\_cp}} w_{qplatc} * PScore \quad (11)$$

$$mATOMS^{TM}Price_{*} - the\ price\ from\ return\ for\ pool\ in\ ATOMS$$

$$w_{platc} = \frac{currBal}{\sum_{platc \in POOL_{ag\_tm\_cp}} currBal} \quad (12)$$

$$mATOMS^{TM}Price_{ag\_tm\_cp} = \sum_{platc \in POOL_{ag\_tm\_cp}} w_{platc} * mATOMS^{TM}Price \quad (13)$$

$$ratio = \frac{PScore}{aggPScore_{ag\_tm\_cp}} \quad (14)$$

$$tTSP^{TM}Price = mATOMS^{TM}Price_{ag\_tm\_cp} * exp^{sratio*ratio-ratio*pow(ratio,xratio)} \quad (15)$$

$Where\ the\ sratio\ is\ the\ following\ table$:

	G1 15	*G2 15*	*FN 15*	*FH 15*	*G1 30*	*G2 30*	*FN 30*	*FH 30*
2	0.005	0.05	0.005	0.005	0.005	0.005	0.005	0.005
2.5	0.15	0.08	0.008	0.03	0.005	0.005	0.12	0.005
3	0.2	0.04	0.0075	0.005	0.04	0.002	0.08	0.03
3.5	0.15	0.04	0.035	0.05	0.038	0.037	0.08	0.04
4	0.12	0.3	0.09	0.1	0.15	0.06	0.026	0.09
4.5	0.1	0.3	0.07	0.12	0.075	0.2	0.08	0.036
5	0.15	0.005	0.05	0.09	0.145	0.2	0.2	0.045
5.5	0.075	0.005	0.005	0.02	0.005	0.2	0.14	0.005
6	0.15	0.005	0.005	0.02	0.005	0.005	0.005	0.06
6.5	0.15	0.005	0.005	0.005	0.005	0.008	0.005	0.005
7	0.15	0.005	0.005	0.005	0.005	0.005	0.005	0.005

Table 1

$Where\ the\ xraito\ is\ the\ following\ table$:

	G1 15	*G2 15*	*FN 15*	*FH 15*	*G1 30*	*G2 30*	*FN 30*	*FH 30*

2	0	0	0	0	0	0	0	0
2.5	0	0	0	0	0	0	0.25	0
3	0	0	0	0	0.35	0	0.5	0
3.5	0	0	0.25	0.1	0	0.2	0.6	0
4	0	0	0.35	0.1	0.15	0.1	0.1	0.19
4.5	0	0	0	0	0.15	0.5	0	0
5	0	0	0	0.2	0	0	0.35	0
5.5	0	0	0	0.6	0	0	0	0
6	0	0	0	0	0	0	0	0.0001
6.5	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

Table 2

$$TSP^{TM}Price = \frac{tTSP^{TM}Price}{exp^{(1-currFactor)*factorRatio}} \quad (16)$$

$$w_{pr} = \frac{currBal}{\sum_{pr\in TSP} currBal} \quad (17)$$

$$TSP^{TM}IndexPrice = \sum_{pr\in TSP} w_{pr} * TSP^{TM}Price \quad (18)$$

$Where\ the\ factorRatio\ is\ the\ following\ table$:

	G1 15	*G2 15*	*FN 15*	*FH 15*	*G1 30*	*G2 30*	*FN 30*	*FH 30*
2	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
2.5	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002
3.5	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001
4	0.002	0.001	0.001	0.015	0.001	0.001	0.001	0.001

4.5	0.002	0.001	0.0065	0.009	0.001	0.005	0.0045	0.001
5	0.001	0.001	0.002	0.005	0.002	0.005	0.001	0.001
5.5	0.001	0.001	0.001	0.003	0.005	0.001	0.001	0.001
6	0.001	0.001	0.001	0.015	0.005	0.001	0.001	0.001
6.5	0.001	0.001	0.001	0.001	0.005	0.001	0.001	0.003
7	0.001	0.001	0.001	0.001	0.005	0.001	0.001	0.001

Table 3

* See US patent: Unified ATOMS™ Index Return and Price Algorithms. Patent application # 62482281

DESCRIPTION TRADEABLE SPECIFIED POOL INDEX RETURN

$$mtdMktReturn = \frac{TSP^{TM}Price - baseTSP^{TM}Price}{baseTSP^{TM}Price + \frac{coupon}{12}} \quad (19)$$

Where

$baseTSP^{TM}Price\ is\ the\ TSP^{TM}Price\ of\ last\ business\ day\ of\ previous\ month$

$$cashAccr = (DAY(priceDate) - 1) * \frac{coupon}{360} \quad (20)$$

$$couponValue = cashAccr + \frac{\frac{coupon}{12}}{(1 + 1mLibor/360)^{daysToCurrPmt}} - \frac{coupon}{12} \quad (21)$$

$$mtdCouponReturn = \frac{couponValue}{baseTSP^{TM}Price + \frac{coupon}{12}} \tag{22}$$

$$paydownValue = \frac{(1 - survFactor) * (100 - TSP^{TM}Price - cashAccr)}{(1 + 1mLibor/360)^{daysToCurrPmt}} \tag{23}$$

$$mtdPaydownReturn = \frac{paydownValue}{baseTSP^{TM}Price + \frac{coupon}{12}} \tag{24}$$

$$mtdTotalReturn$$

$$= mtdMktReurn + mtdPaydownReturn$$

$$+ mtdCouponReturn \tag{25}$$

$$aggReturn = \sum_{pool \in TSP} w_{pool} * mtdTotalReturn_{pool} \tag{26}$$

Where

$$w_{pool} = \frac{baseBal * \left(baseTSP^{TM}Price + \frac{coupon}{12}\right)}{\sum_{pool \in TSP} baseBal * \left(baseTSP^{TM}Price + \frac{coupon}{12}\right)} \tag{27}$$

$$mtdIndexReturn = 1 + aggReturn \tag{28}$$

$$TSP^{TM}IndexReturn = mtdIndexReturn * baseCumulativeIndex \tag{29}$$

where

$baseCumulativeIndex$ is the $TSP^{TM}IndexReturn$ of last business day of previous month.

CLAIMS

A method of constructing Tradeable Specified Pool Index™ and Specified Pool Index™ futures contract.

Figure 1: Spacetime Relationship of Tradeable Specified Pool Index

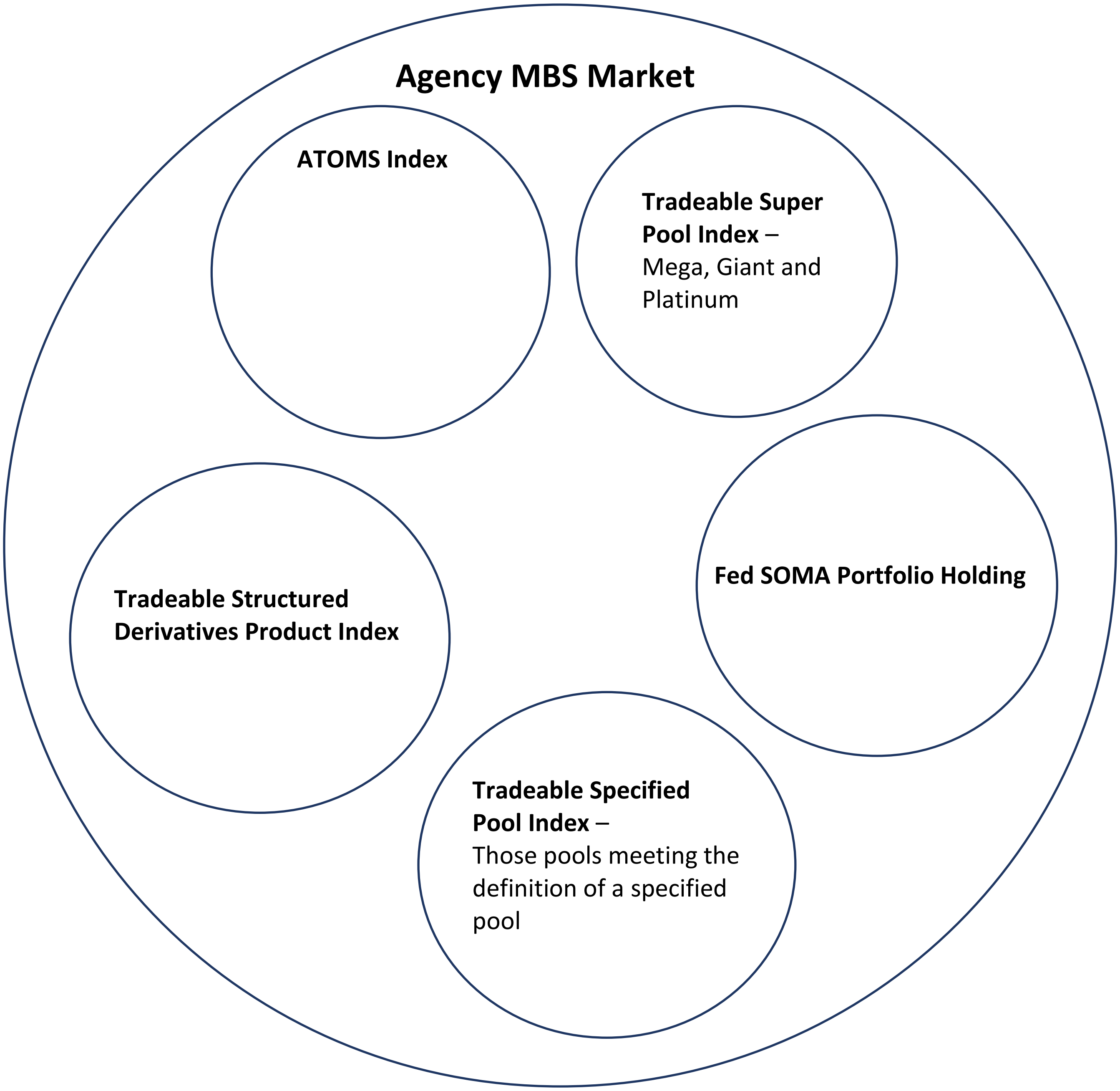
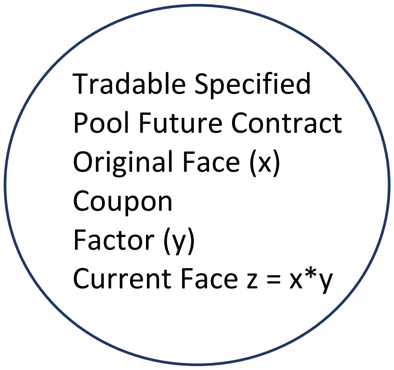
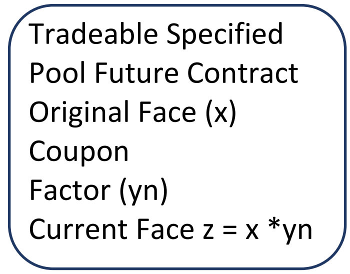


Figure 2: Spacetime Relationship of Tradeable Specified Pool Future Contract

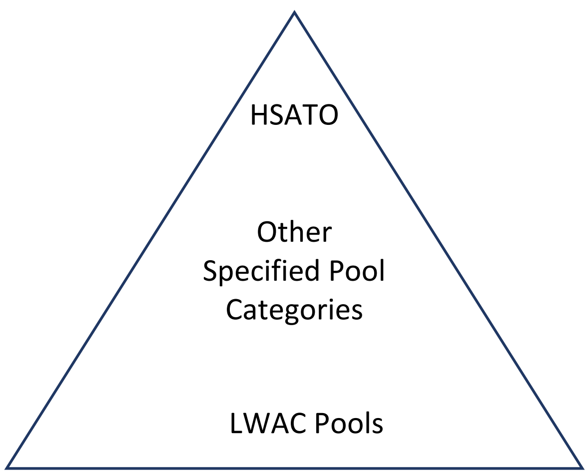




The contract is opened at t_0 mortgage pay-downs reduce the factor over time. The contract is settled based on the [price difference * the factor difference * current face] amount

A method of hierarchy categorization based on the convexity of the specified pool as given by R-score and D-score

Figure 3: Tradeable Specified Pool Index Hierarchy of Categorization



Hierarchy of Categorization – it is possible that a pool may fit into more than one categorizations. The constituents index specified pool is assigned in the reverse waterfall priority as outlined below:

1. LWAC
2. PR 150
3. Puerto Rico
4. Low Loan Balance
5. Moderate Loan Balance
6. HLTV2
7. NY 150
8. FL 150
9. TX 150
10. LA 150
11. LFICO_HLB
12. High Loan Balance
13. HLTV
14. High Loan-to-Value
15. HHLTV
16. LFICO_HLTV
17. LFICO
18. LFICO2
19. Investor
20. New York
21. FL
22. TX-LWAC
23. Texas
24. Illinois
25. NJ-TX-PA
26. NY-TX-FL
27. TX-FL
28. HSATO

A model for forecasting the system performance of the Specified Pool Index™.

A method using deconvolution of space-time dimensions to allow the interaction of dimensions to be included in an individually weighted and for some cases non-linear weighted manner.

A method to calculate the manifold vectors as the sum of dimensions and interaction terms.

A method to calculate the R-score based on the average of the manifold vector over time.

A method to calculate the D-score based on the average of a manifold vector over time.

A method to calculate the QED field effect diffusion process used in the Lie group deconvolution model.

ABSTRACT

The unified QED field effect deconvolution approach is used to compute Tradeable Specified Pool Index R&D risk scores. This approach uses very

large heterogeneous historical loan-level and pool-level databases to calibrate the model. It utilizes innovative unified deconvolution computing of the various space-time dimensional factors so that the individual factor interactions can be included in an individually weighted non-linear manner. The model uses an innovative non-linear calibration to map all scores between 100 and 999 to address 10^{27} combinatorial permutations of interactions in the Tradeable Specified Pool Index™.